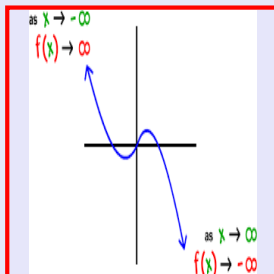


Math 245
Spring 2022
Lecture 35



Given $P(x) = x^3 - 2x^2 - 13x - 10$

1) How many Zeros? why?

at most 3, Degree=3

2) List of all possible rational Zeros: $\frac{\pm \text{All Factors of } 10}{\pm \text{All Factors of } 1}$

$$\frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1} = \pm 1, \pm 2, \pm 5, \pm 10$$

Upper Bound: It is a positive number such that after synthetic division, all numbers in the bottom row are all positive.

there is no Zeros greater than the upper bound.

Is 5 an upper bound?

$$\begin{array}{r|rrrr} 5 & 1 & -2 & -13 & -10 \\ & & 5 & 15 & 10 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

All positive

5 is a Zero as well.

Positive

5 is an upper bound.

There are no Zeros greater than 5.

Lower Bound: It is a negative number such that the numbers in the bottom row of synthetic division have alternating signs.

Is -5 a lower bound?

$$\begin{array}{r|rrrr} -5 & 1 & -2 & -13 & -10 \\ & & -5 & 35 & -110 \\ \hline & 1 & -7 & 22 & -120 \end{array}$$

Negative Number
 Alternating Signs
 -5 is a lower bound
 there are no zeros less than -5.

No Zeros -5 Possible Zeros 5 No Zeros

$$\begin{array}{r|rrrr} 5 & 1 & -2 & -13 & -10 \\ & & 5 & 15 & 10 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

is -2 a zero? Yes

$$\begin{array}{r|rr} -2 & 1 & 3 & 2 \\ & & -2 & -2 \\ \hline & 1 & 1 & 0 \end{array}$$

$$P(x) = (x-5)(x^2 + 3x + 2)$$

$$= (x-5)(x+2)(x+1)$$

$$P(x) = (x-5)(x+2)(x+1)$$

Zeros 5, -2, -1

$$P(x) = 6x^3 + 17x^2 - 31x - 12$$

At most 3 Zeros

Possible Rational Zeros $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
 $\pm 1, \pm 2, \pm 3, \pm 6$

~~$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{2}{3}, \pm \frac{3}{2}, \pm \frac{4}{3}, \pm \frac{6}{5}, \pm \frac{12}{5}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{3}{3}, \pm \frac{4}{3}, \pm \frac{6}{3}, \pm \frac{12}{3}, \pm \frac{1}{6}, \pm \frac{2}{6}, \pm \frac{3}{6}, \pm \frac{4}{6}, \pm \frac{6}{6}, \pm \frac{12}{6}$~~

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}$

Is 4 an upper bound?

$$\begin{array}{r|rrrr} 4 & 6 & 17 & -31 & -12 \\ & & 24 & 164 & + \\ \hline & 6 & 41 & 133 & + \end{array}$$

4 is an upper bound.

Is 2 an upper bound?

$$\begin{array}{r|rrrr} 2 & 6 & 17 & -31 & -12 \\ & & 12 & 58 & 54 \\ \hline & 6 & 29 & 27 & + \end{array}$$

2 is an upper bound.

is -2 a lower bound?

$$\begin{array}{r} -2 \overline{) 6 \quad 17 \quad -31 \quad -12} \\ \underline{ 12 \quad -10 \quad 82} \\ 6 \quad 5 \quad -41 \quad 70 \end{array}$$

NO, Signs are not alternating in the bottom row.

Is -3 a lower bound?

$$\begin{array}{r} -3 \overline{) 6 \quad 17 \quad -31 \quad -12} \\ \underline{ 18 \quad 3} \\ 6 \quad -1 \quad -28 \end{array}$$

NO,

Is -5 a lower bound?

$$\begin{array}{r} -5 \overline{) 6 \quad 17 \quad -31 \quad -12} \\ \underline{ 30 \quad 65 \quad -170} \\ 6 \quad -13 \quad 34 \quad -182 \end{array}$$

Yes, Signs are alternating in the bottom row.

	LB		UB	
NO	-5	Possible	2	NO
Zeros		Zeros		Zeros

Determine Show $\frac{1}{2}$ is a Zero of $P(x)$.

$$\begin{array}{r} \frac{1}{2} \overline{) 6 \quad 17 \quad -31 \quad -12} \\ \underline{\phantom{\frac{1}{2}} 3 \quad 10 \quad -2\frac{1}{2}} \\ 6 \quad 20 \quad -21 \quad \neq 0 \text{ Not Zero} \end{array}$$

$\frac{1}{2}$ is not a Zero.

what about $\frac{3}{2}$?

$$\begin{array}{r} \frac{3}{2} \overline{) 6 \quad 17 \quad -31 \quad -12} \\ \underline{\phantom{\frac{3}{2}} 9 \quad 39 \quad 12} \\ 6 \quad 26 \quad 8 \quad 0 \end{array}$$

$\frac{3}{2}$ is a Zero. Rem. = 0

$P(x) = (x - \frac{3}{2})(6x^2 + 26x + 8)$
 $= (x - \frac{3}{2}) \cdot 2(3x^2 + 13x + 4)$
 $P(x) = (2x - 3)(3x^2 + 13x + 4)$

Is -4 a Zero? $\begin{array}{r} -4 \overline{) 3 \quad 13 \quad 4} \\ \underline{ 12 \quad -12 \quad 4} \\ 3 \quad 1 \quad 0 \end{array}$

Yes, Rem. = 0

$P(x) = (2x - 3)(3x + 4)(x - (-4))$
 $= (2x - 3)(3x + 4)(x + 4)$

Zeros $\Rightarrow \frac{3}{2}, -\frac{4}{3}, -4$

	LB	UB
	-5	2

$$P(x) = 6x^4 + 2x^3 + 9x^2 + x + 5$$

1) How many Zeros? Why?

At most 4, Degree=4

2) Is 1 an upper bound? why?

1	6	2	9	1	5	Yes, All Signs are +.
		6	8	17	18	
	6	8	17	18	23	

3) Is -1 a lower bound? why?

-1	6	2	9	1	5	Yes, Signs are Alternating
		-6	4	-13	12	
	6	-4	13	-12	17	

